

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
NUMBER

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

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1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places.

[3]

[illegible]

- 2 (a)** Expand $\sqrt[3]{1+6x}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** State the set of values of x for which the expansion is valid. [1]

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- 3** The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

- (a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Find the exact x -coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

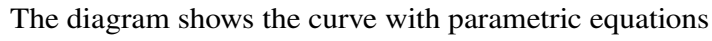
- 4 (a)** Show that the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$ can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0. \quad [3]$$

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

- (b) Hence solve the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$, for $0^\circ < \theta < 180^\circ$. [3]

[illegible]



for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

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- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

The gradient of the curve has its maximum value at the point P .

(b) Find the exact value of the x -coordinate of P . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 The complex number u is defined by

$$u = \frac{7+i}{1-i}.$$

- (a) Express u in the form $x + iy$, where x and y are real. [3]

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- (b) Show on a sketch of an Argand diagram the points A , B and C representing u , $7 + i$ and $1 - i$ respectively. [2]

$$\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi. \quad [3]$$

[illegible]

7 The variables x and t satisfy the differential equation

$$e^{3t} \frac{dx}{dt} = \cos^2 2x,$$

for $t \geq 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t . [7]

[illegible]

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(b) State what happens to the value of x when t tends to infinity.

[1]

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- 8 With respect to the origin O , the position vectors of the points A , B , C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that $AB = 2CD$.

[3]

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- (b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

[3]

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- (c) Show that the line through A and B does not intersect the line through C and D . [4]

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9 Let $f(x) = \frac{7x+18}{(3x+2)(x^2+4)}$.

(a) Express $f(x)$ in partial fractions.

[5]

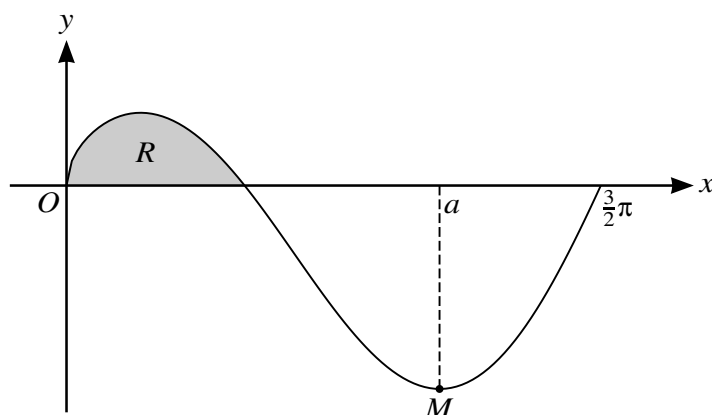
[illegible]

(b) Hence find the exact value of $\int_0^2 f(x) \, dx$.

[6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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The diagram shows the curve $y = \sqrt{x} \cos x$, for $0 \leq x \leq \frac{3}{2}\pi$, and its minimum point M , where $x = a$. The shaded region between the curve and the x -axis is denoted by R .

- (a) Show that a satisfies the equation $\tan a = \frac{1}{2a}$. [3]

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- (b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\frac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a .

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (c) Find the volume of the solid obtained when the region R is rotated completely about the x -axis. Give your answer in terms of π . [6]

[illegible]

[illegible]

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